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LINEAR ALGEBRA & MATRIX IN MATHEMATICS: UNDERSTANDING IMPORTANCE

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Abstract:
In linear algebra, we learn the basics of linear systems by studying matrices and vectors. A branch of mathematics known as linear algebra primarily studies vectors, vector spaces (sometimes called linear spaces), linear mappings (often called transformations), and systems of linear equations.
Since vector spaces are fundamental to contemporary mathematics, abstract algebra and functional analysis both make extensive use of linear algebra. Analytic geometry provides a tangible example of linear algebra, while operator theory provides a generalization of the theory. It finds widespread use in the social and natural sciences due to the fact that linear models may often be used to approximate nonlinear ones.
Keywords: Linear Equation, Linear Spaces, n-Tuples, Matrix, and Linear Algebra.

I. INTRODUCTION

The foundations of linear algebra were laid by studies of vectors in Cartesian two- and three-space. To clarify, in this specific situation, a vector is a coordinated line portion that is defined by its direction and magnitude, which are measured in length. One possible representation of physical things is a vector, which, when combined with other vectors and multiplied by scalars, may be seen as the first real vector space. New developments in modern linear algebra allow for the inclusion of spaces with any number of dimensions, up to infinity. The abbreviation "n-space" describes a vector space with n dimensions. It is possible to generalize most of the significant findings from two- and three-dimensional spaces to these higher-dimensional ones. To represent data, n-tuples or vectors are helpful, even if they are difficult for humans to see in n-space. Vectors, being ordered lists of n components, provide for efficient data summarization and manipulation in this system. In economics, one example is the creation and use of 8-dimensional vectors or 8-tuples to represent the Gross National Product of eight countries. Using a vector (v1, v2, v3, v4, v5, v6, v7, v8) with the GNP of each country in its own location, you may select to display the gross national product (GNP) of eight nations for a particular year in a certain order, such as: (US, UK, France, Germany, Spain, India, Japan, Australia). Since it is a completely abstract notion over which theorems are proven, vector spaces (also known as linear spaces) are an integral aspect of abstract algebra. The ring of vector space linear mappings and the group of invertible linear maps or matrices are two well-known examples of this. The study of alternating maps and tensor products, as well as the explanation of higher-order derivatives in vector analysis, are other areas where linear algebra plays an important role in analysis.

The investigation of vectors in cartesian 2-space and 3-space was the first area of direct polynomial math [Rathee, S. (2023)]. In this specific circumstance, a vector is a coordinated portion of a line that is characterized by its heading and size, which are alternative names for the same thing. A vector that is neither positive nor negative in magnitude nor direction is known as a zero vector. This is the earliest known instance of a real vector space, where the "scalars" are real numbers and the "vectors" are physical phenomena like forces; the vectors may be multiplied by scalars and combined together to make the space.(Spurio, 2023)'s work
Many mathematicians regard vectors to be among the most abstract concepts (Aguirre and Erickson, 1984; Knight, 1995; Poynter and Tall, 2005). Not only is it essential for 21st century technological growth, but it is also foundational to linear algebra (in areas like vector spaces, linear combinations, linear transformations, eigenvalues, and eigenvectors) (Stewart et al., 2019). Spaces with arbitrary or infinite dimensions are now within the purview of modern linear algebra. A space with n dimensions is known as an n-space vector space. The majority of the practical outcomes from two- and three-

dimensional spaces may be carried over to these higher-dimensional ones. Vectors or n-tuples are valuable data representations, even if humans struggle with seeing them in n-space. This framework is great for summarizing and manipulating information since vectors, as n-tuples, have n requested parts. In economics, for instance, it is possible to construct and utilize 8-layered vectors or 8-tuples to represent the Gross domestic product of eight unique countries. You can decide to show the gross public item (GNP) of eight nations for a given year in a particular request, similar to this: (US, UK, Armenia, Germany, Brazil, India, Japan, Bangladesh) using a vector (v1, v2, v3, v4, v5, v6, v7, v8) with the GNP of each country in its own position.

1.1 SOME USEFUL THEOREMS

According to Korevaar (2014) and Anton (1985), each vector space has its own premise.

The element of a vector space is obvious, implying that any two bases of a similar space have a similar cardinality.

The sole condition for a matrix to be invertible is for its determinant to be nonzero. Source: Singh (2021) In the event that the straight guide that a framework addresses is an isomorphism, the matrix may be inverted.

See invertible matrix for additional comparable expressions; in the event that a square lattice has an opposite on the left or the right, it is invertible.

When all of a matrix's eigenvalues are positive or equal to zero, we say that the matrix is positive semidefinite.

Each eigenvalue of a positive definite matrix must be bigger than zero for the matrix to be considered positive definite.

If an n×n matrix has n directly autonomous eigenvectors, then it tends to be diagonalized, meaning that it can be transformed into an invertible network P and a corner to corner lattice D to such an extent that A = PDP-1.

A symmetric matrix is the only one that may be orthogonally diagonalizable, according to the spectral theorem.

2 Linear Equation

Algebraic equations where each term is a consistent or the result of a steady and (the principal force of) a solitary variable are called straight conditions. You might have at least one factors in a direct condition. Direct conditions are universal in the field of science and its numerous subfields, especially in applied math. The suspicion that upsides of interest change to an insignificant level from some "foundation" condition reduces many non-linear equations to linear equations, which are very helpful. These equations appear naturally when modeling numerous events. In linear equations, exponents are not considered. In this piece, we play the role of a researcher looking for the actual answers to a single equation. Not only does it include complicated solutions, but it also covers straight conditions with coefficients and arrangements in any field in a more broad sense.

3 Matrix

The rectangular array of numerical values (or other mathematical objects) that make up a whole matrix is referred to as its entries. According to Bernstein (2009), matrices may be easily transformed using common mathematical operations like addition and multiplication. As an example, this matrix is based on reality:

$$\mathbf{A} = \begin{bmatrix} -1.3 & 0.6 \\ 20.4 & 5.5 \\ 9.7 & -6.2 \end{bmatrix}$$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array}$$

Figure 1: A m*n matrix(source: Wikipedia)

As seen on the right, a matrix (plural matrices, or less often just matrices) is a numerical array laid out in a rectangular fashion.

Tables containing numerical data derived from physical observations are a common place to see matrices, but you may also find them in other mathematical settings. For instance, as we'll learn in the next chapter, all the necessary data to resolve an arrangement of conditions, such

$5x + y = 3$

$2x - y = 4$

embedded in the matrix, and that the system's solution may be derived by applying the right operations to this matrix.

$$\begin{array}{ccc} 5 & 1 & 3 \\ 2 & -1 & 4 \end{array}$$

Tensors are arrays of numbers with more dimensions than one dimension, such as three dimensions, whereas vectors are matrices with only one column or row. It is possible to multiply matrices according to a method that compares to the creation of straight changes, and to add and remove them entry by entry. Except for the fact that matrix multiplication is not commutative, these operations meet the standard identities; nonetheless, the identity AB=BA might fail. The representation of linear transformations in matrices is one use case. These transformations are higher-dimensional versions of the same linear functions, where c is a constant and f(x) = cx. Another useful use of matrices is to record the values of the coefficients in a set of direct conditions. Settling an arrangement of straight conditions with a square lattice is constrained by the determinant and, if present, the opposite network. The geometry of the related linear transformation may be understood from the eigenvalues and eigenvectors. There are a lot of uses for matrices. Several areas of physics make use of them, including matrix mechanics and geometrical optics.

3.1 Addition and Subtraction of linear matrices

Matrix A and B's sum (difference) is the matrix that results from adding (subtracting) the elements at corresponding positions of A and B. Since the rules for adding and subtracting linear matrices only apply to matrices of equal order,

Thus,

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 3 & -1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 3 & -3 \end{bmatrix}$$

$$\Rightarrow \quad A + B = \begin{bmatrix} 0 & 6 & 5 \\ 7 & 2 & -3 \end{bmatrix} \text{ and } A - B = \begin{bmatrix} 2 & 2 & -1 \\ -1 & -4 & 3 \end{bmatrix}.$$

However, if

$$C = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix},$$

Consequently, C is an element that cannot be combined with A or B in any way.

As a fundamental concept in linear algebra, matrices are closely related to linear transformations. In addition to rings and elements from more broad mathematical domains, there are other sorts of entries that are used.

3.2 Transformation of Matrices

As an example, a 2*1 column vector may be obtained by premultiplying it with a 2*2 matrix:

$\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 17 \\ -9 \end{bmatrix}$

If the vector $\begin{bmatrix} 7 \\ -1 \end{bmatrix}$ The matrix has transformed the point with coordinates (7, -1) into the new point with coordinates (17, -1), which is represented by the position vector I. In a similar vein, the matrix influences every single point on the plane. This may be expressed as T, the transformation,

$T = \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x' \\ y' \end{bmatrix}$

Image points (x', y') are transformed into point (x, y) using T.
Based on the matrix provided

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x + & 4y \\ -x + & 2y \end{bmatrix}$$

Additionally, the transformation may be expressed in the following way:
T: x'= 3x+4y, y'= -x+2y.

3.3 Matrix multiplication

The conditions for the definition of matrix multiplication are that the left matrix must have an equal number of segments and the right lattice should have an equivalent number of lines. When An and B are m-by-n and n-by-p lattices, individually, the m -by-p matrix AB is obtained by multiplying them. The elements of this matrix are supplied by: where $1 \leq i \leq m$ and $1 \leq j \leq p$.[5]
We must consider the vector as a column matrix in order to specify the matrix-vector product, which is the multiplication of a matrix A with a vector x. The network vector item is characterized exclusively for the circumstance when the quantity of sections in A is equivalent to the quantity of lines in x. Thus, for $n\times 1$ column vectors x, the product Ax is defined if A is a $m\times n$ matrix (i.e., with n columns). The vector b is a $m\times 1$ column vector if we let $Ax=e$. Put otherwise, the product of b and A is characterized by the quantity of lines in A, which might be anything.
One common way to calculate the matrix-vector product is

$$A\mathbf{x} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

The procedure of matrix-vector multiplication is really very straightforward, despite its seeming complexity. For each row of A, one obtains the dot product of x. For this reason, it is essential that the quantity of sections in A be equivalent to the quantity of parts in x. In the network vector item, the initial segment is the spot result of x and the main line of **s**,and so on. really, the matrix-vector product is really just a dot product masquerading as A if A has a single row.

3.4 Linear Equations

Babylonians, some 4,000 years ago, could solve a basic system of two-by-two linear equations involving just two variables. An ability to tackle a 3x3 arrangement of conditions was shown by the Chinese in 200 BC when they produced "Nine Chapters of the Mathematical Art" (Perotti). As far back as recorded history goes, individuals from all walks of life have attempted to solve the seemingly easy

equation ax+b=0. Late in the 17th century was when linear algebra's strength and advancements were realized.
Turn of the nineteenth century saw the introduction of a technique for tackling frameworks of straight conditions via Carl Friedrich Gauss [Ferreirós, J. (2020)]. While he did touch on linear equations in his writings, matrices and their notations were still outside of his purview. Different equations involving different numbers and variables were the focus of his study, along with the classics from the pre-modern era by Euler, Leibnitz, and Cramer. To summarize Gauss's work, the phrase "Gaussian elimination" is now used. Some equations may have their variables removed using this technique, which relies on the ideas of merging, exchanging, or multiplying rows with each other. Once the variables have been discovered, the remaining unknown variables may be found by using back substitution [Ali et. al., 2021].
An illustration of grid increase that is firmly connected with straight conditions is the point at which An is a m-by-n framework and x is a segment vector (i.e., a n×1-lattice) with n factors x1, x2,..., xn. For this situation, the grid condition Hatchet = b, where b is a m×1-section vector, is equivalent to the arrangement of straight conditions (Bronson, Richard, 1989).

$$A_{1,1}x_1 + A_{1,2}x_2 + \ldots + A_{1,n}x_n = b_1$$
$$A_{m,1}x_1 + A_{m,2}x_2 + \ldots + A_{m,n}x_n = b_m$$

Matrix algebra allows for the concise representation and handling of frameworks of straight conditions, which comprise of a few direct conditions.

4 Conclusion
Contemporary physics relies heavily on linear transformations and the corresponding symmetries. The use of quantum theory to the study of chemical bonding and spectroscopy has greatly expanded the range of applications for matrices in chemistry. We provide here a mathematical study of matrices and linear algebra.
When an algebraic equation has terms that are either constants or the product of a constant and one variable raised to the power of one, we say that the equation is linear. It is possible for linear equations to have many variables. Linear algebra delves into a set of linear equations, vector spaces (sometimes called linear spaces), linear mappings (also called linear transformations), and vectors.

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